# A model for the microwave assisted zero resistance states

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**Abstract**

In this work we present a model for the photoconductivity of a two dimensional electron system (2DES) subjected to a magnetic field. The model includes the microwave and Landau contributions in a non-perturbative exact way, Impurity scattering effects are treated perturbatively. Based on this formalism, we provide a Kubo-like formula that takes into account the oscillatory Floquet structure of the problem. We discuss results related with the recently discovered zero-resistance states.

## 1 Introduction.

Recently, two experimental groups [1, 2, 3, 4], reported the observation of a novel phenomenon: the existence of zero-resistance states in an ultraclean $GaAs/Al_xGa_{1-x}As$ sample subjected to microwave radiation and moderate magnetic fields. The magnetoresistance exhibits strong oscillations with regions of zero resistance governed by the ratio $\omega/\omega_c$, where $\omega_c$ is the cyclotron frequency. According to Zudov *etal.*, the oscillation amplitudes reach maxima at $\omega/\omega_c = j$ and minima at $\omega/\omega_c = j + 1/2$, for $j$ an integer. On the other hand Mani. *etal.* reported also a periodic oscillatory behavior, but with maxima at $\omega/\omega_c = j - 1/4$ and minima at $\omega/\omega_c = j + 1/4$.

In spite of a large number of theoretical works, a complete understanding has not yet been achieved. A pioneer work put forward by Ryzhii [5, 6] predicted the

existence of negative-resistance states. Durst and collaborators [7] also found negative-resistance states in a a diagrammatic calculation of the photoexcited electron scattered by a disorder potential. A possible connection between the calculated negative-resistance states and the observed vanishing resistance was put forward in reference [8], noting that a general analysis of Maxwell equations shows that negative resistance induces an instability that drives the system into a zero-resistance state. In our model, the Landau-Floquet states act coherently with respect to the oscillating field of the impurities, that in turn induces transitions from levels below Fermi to levels above it. This formalism is complemented with a generalization of the Kubo formalism in order to correctly include the Floquet structure of the problem.

# 2 The model.

We consider the motion of an electron in two dimensions, subject to a uniform magnetic field $\mathbf{B}$ perpendicular to the plane and driven by microwave radiation. In the long-wave limit the dynamics is governed by the Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi = \left[H_{\{B,\omega\}} + V(\boldsymbol{r})\right]\Psi\,, \tag{1}$$

here $H_{\{B,\omega\}}$ is the Landau hamiltonian coupled to the radiation $H_{\{B,\omega\}} = \frac{1}{2m^*}\mathbf{\Pi}^2$, $m^*$ is the effective electron mass, $\mathbf{\Pi} = \mathbf{p} + e\mathbf{A}$, and the vector potential $\mathbf{A}$ includes the external magnetic field and radiation field (in the $\lambda \to \infty$ limit) contributions: $\mathbf{A} = -\frac{1}{2}\boldsymbol{r}\times\boldsymbol{B} + Re\,\left[\frac{\boldsymbol{E}}{\omega}\exp\{-i\omega t\}\right]$. The impurity scattering potential is decomposed in a Fourier expansion $V(\boldsymbol{r}) = \sum_i \int d^2q\, V(\boldsymbol{q})\exp\{i\boldsymbol{q}\cdot(\boldsymbol{r}-\boldsymbol{r}_i)\}$, where $\boldsymbol{r}_i$ is the position of the $i$th impurity, and the explicit form of the potential coefficient $V(\boldsymbol{q})$ for neutral impurity scattering is $V(\boldsymbol{q}) = \frac{2\pi^2\hbar^2 V_0}{m^*\mathcal{E}_F}$, whereas for charged impurities localized within the doped layer of thickness $d$: $V(\vec{q}) = \frac{\pi\hbar^2}{m^*}e^{-qd}/\left(1+\frac{q}{q_{TF}}\right)$, with $q_{TF} = e^2 m^*\left(2\pi\epsilon_0\epsilon_b\hbar^2\right)$.

A three step procedure is enforced in order to solve the problem posed by eq. (1): (1) The Hamiltonian $H_{\{B,\omega\}}$ can be exactly diagonalized by a transformation of the form $W^\dagger H_{\{B,\omega\}} W = \omega_c\left(\frac{1}{2} + a_1^\dagger\, a_1\right) \equiv H_0$, with the $W(t)$ operator given by

$$W(t) = \exp\{i\eta_1 Q_1\}\exp\{i\xi_1 P_1\}\exp\{i\eta_2 Q_2\}\exp\{i\xi_2 P_2\}\exp\{i\int^t \mathcal{L}dt'\}, \tag{2}$$

where the functions $\eta_i(t)$ and $\xi_i(t)$ represent the solutions to the classical equations of motion and the $(Q_\mu, P_\mu)$ operators are the generators of the electric magnetic translation symmetries [9, 10]. (2) The transformation induced by $W$ is applied to the Schrödinger eq. (1), transformed into $i\hbar\frac{\partial\Psi^{(W)}}{\partial t} = (H_0 + V_W(t))\,\Psi^{(W)}$, where $V_W(t) = W(t)V(\boldsymbol{r})W^{-1}(t)$ and $\Psi^{(W)} = W(t)\Psi$. Note that the impurity potential acquires a time dependence brought by the $W(t)$ transformation. (3) The problem is now solved in terms of an evolution

operator $U(t)$, using the interaction representation and first order time dependence perturbation theory. The solution to the original Schrödinger equation in eq. (1) has been achieved by means of three successive transformations

$$|\Psi_\mu(t)\rangle = W^\dagger \, \exp\{-iH_0 t\} \, U(t-t_0) \, |\mu\rangle. \tag{3}$$

The explicit expressions for the matrix element of these operators in the Landau-Floquet base appear in detail in reference [11].

# 3 Kubo formula for Floquet states.

The usual Kubo formula for the conductivity must be modified in order to include the Floquet dynamics. In the presence of an additional $DC$ electric field the complete Hamiltonian is $H_T = H + V_{ext}$, where $H$ is the Hamiltonian in eq. (1) and $V_{ext} = \frac{1}{m}\mathbf{\Pi} \cdot \boldsymbol{A}_{ext}$ with $\boldsymbol{A}_{ext} = \frac{\boldsymbol{E}_0}{\omega}\, sin\,(\Omega t)\; exp\,(-\eta|t|)$. The static limit is obtained with $\Omega \to 0$, and $\eta$ represents the rate at which the perturbation is turned on and off. In order to calculate the expectation value of the current density, we need the density matrix $\rho(t)$ which obeys the von Neumann equation

$$i\hbar \frac{\partial \rho}{\partial t} = [H_T, \rho] = [H + V_{ext}, \rho]\,. \tag{4}$$

We write to first order $\rho = \rho_0 + \Delta\rho$, where the density matrix $\rho_0$ satisfies the equation $i\hbar\frac{\partial \rho_0}{\partial t} = [H, \rho_0]$. In agreement with eq. (3) the density matrix is transformed as $\tilde{\Delta}\rho(t) = U^\dagger \exp\{iH_0 t\} W \Delta\rho(t) W^\dagger \exp\{-iH_0 t\} U$, and obeys the equation $i\hbar\frac{\partial \tilde{\Delta}\rho}{\partial t} = \left[\tilde{V}_{ext}, \tilde{\rho}_0\right]$, where $\tilde{V}_{ext}$ and $\tilde{\rho}_0$ are the external potential and quasi-equilibrium density matrix transformed in the same manner as $\tilde{\Delta}\rho$. The transformed quasi-equilibrium density matrix is assumed to have the form $\tilde{\rho}_0 = \sum_\mu |\mu\rangle f(\epsilon_\mu) \langle\mu|$, where $f(\epsilon_\mu)$ is the usual Fermi function and $\epsilon_\mu$ the Landau-Floquet levels. The argument behind this selection is an adiabatic assumption that the original Hamiltonian $H$ produces a quasi equilibrium state characterized by the Landau-Floquet eigenvalues. We then obtain the expectation value of $\tilde{\Delta}\rho(t)$ in the Landau-Floquet base, from which the current density is evaluated according to $\langle \boldsymbol{J}(t, \boldsymbol{r})\rangle = Tr\left[\tilde{\Delta}\rho(t)\tilde{\boldsymbol{J}}(t)\right]$. The macroscopic conductivity tensor that relates the average current density to the averaged electric field, is obtained performing a space-time average, as well as an impurity average, assuming non correlated impurities. We take into account the lifetime $\tau = 2\pi/\eta$ of the quasiparticles induced by the weak scattering, in the usual Born approximation. We work out explicit expressions for the longitudinal and Hall conductivities (both for the dark and microwave induced contributions) [11], and quote the result for the longitudinal photoconductivity

$$\langle \boldsymbol{\sigma}_L^\omega \rangle = \frac{\pi e^2 n_I}{\hbar} \int d\epsilon \sum_{\mu\nu} \sum_l A\,(\epsilon - \epsilon_\mu)\, B^{(l)}\,(\epsilon, \epsilon_\nu) \int d^2 q K(\boldsymbol{q}) \Big| J_l\,(|\Delta|)\, V(\boldsymbol{q}) D_{\mu\nu}(\tilde{q}) \Big|^2, \tag{5}$$

where $n_I$ is the two dimensional impurity density, and the broadened spectral function is given as
$A(\epsilon-\epsilon_\mu) = \frac{\imath}{2\pi}\left[G_\mu^+(\epsilon) - G_\mu^-(\epsilon)\right]$, where $G_\mu^\pm(\epsilon) = 1/\left(\epsilon - \epsilon_\mu \pm \imath\eta/2\right)$ are the advanced and retarded Green's functions with finite lifetime $2\eta^{-1}/\hbar$. The auxiliary functions $K(\boldsymbol{q})$ and $B^{(l)}$ are defined according to

$$K(\boldsymbol{q}) = \omega_c^2 l_B^2 \; \frac{q_x^2\left[(\epsilon_{\mu\nu}+\omega l)^2+\eta^2\right] + q_y^2\omega_c^2 - 2\omega_c q_x q_y \eta}{|\,(\epsilon_{\mu\nu}+\omega l - \imath\eta)^2 - \omega_c^2|^2}, \tag{6}$$

and

$$B^{(l)}(\epsilon,\epsilon_\nu) = \left[\frac{d}{d\epsilon_0}\left\{\left[f(\epsilon + l\omega + \epsilon_0) - f(\epsilon)\right] A(\epsilon - \epsilon_\nu + l\omega + \epsilon_0)\right\}\right]_{\epsilon_0=0}. \tag{7}$$

In eq. (5) $J_l\left(|\Delta|\right)$ is the Bessel function with $\Delta = \frac{\omega_c l_B^2 eE}{\omega(\omega^2-\omega_c^2+i\omega\Gamma)}\left[\omega\left(q_x e_x + q_y e_y\right) + i\omega_c\left(q_x e_y - q_y e_x\right)\right]$, $l_B = \sqrt{\frac{\hbar}{eB}}$ being the magnetic length, $\Gamma$ the electron radiative decay width and $\boldsymbol{\epsilon}$ the polarization vector. Finally $D^{\nu\mu}\left(\tilde{q}\right) = e^{-\frac{1}{2}|\tilde{q}|^2}\tilde{q}^{\nu-\mu}\sqrt{\frac{\mu!}{\nu!}}L_\mu^{\nu-\mu}\left(|\tilde{q}|^2\right)$, with $L_\mu^\mu$ being the generalized Laguerre polynomial and $\tilde{q} = il_B(q_x - iq_y)/\sqrt{2}$.

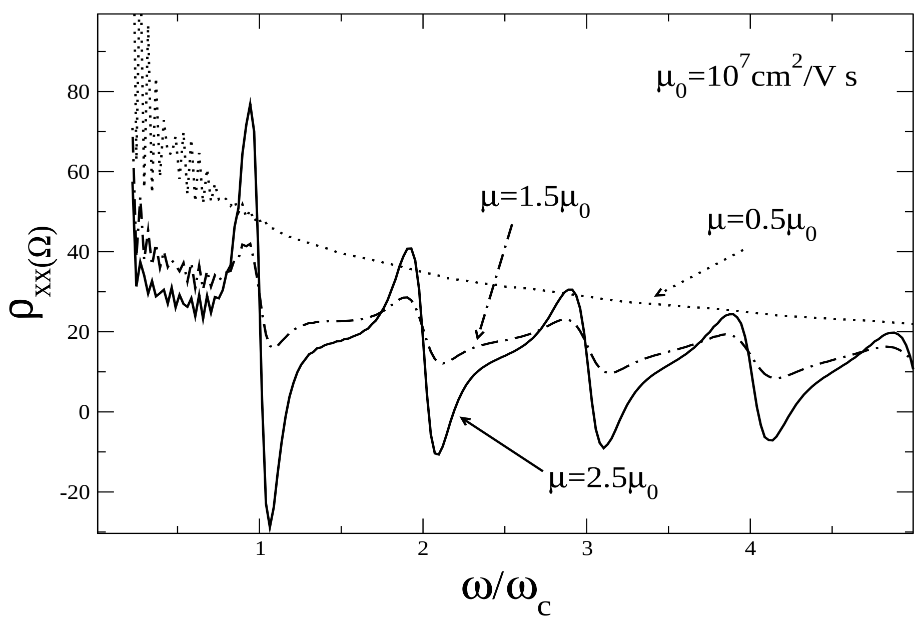


Figure 1: Longitudinal resistance as a function of $\omega/\omega_c$ for three selections of the mobility.

# 4 Results and conclusions

We have selected parameter values corresponding to reported experiments [1, 3]: effective electron mass $m^* = 0.067\, m_e$, relative permittivity $\epsilon_b \approx 13.18$, fermi energy $\epsilon_F = 8\, meV$, electron mobility $\mu \approx 0.5 - 2.5 \times 10^7 cm^2/Vs$, electron density $n = 3 \times 10^{11} cm^{-2}$, microwave frequency $f = 100\, Ghz$, magnetic fields in the range $0.05 - 0.4\ Tesla$ and temperatures $T \approx 1\, K$. Typical microwave power is $10 - 40\, nW$ from which the electric field intensity is estimated as $|\vec{E}| \approx 250\, V/m$. The broadening $\eta$ is known to increase with the square root of the magnetic field, hence we take $\eta^2 = \hbar\omega_c\, (2\pi\hbar/\tau)$, which is connected through the relaxation time $\tau$ with the zero field mobility $\mu = e\tau/m^*$. In the case of charged impurity scattering, the distance $d$ between the impurity and the 2DES is taken as $d \approx 30\, nm$, and $n_I$ is estimated as $n_I \approx 7 \times 10^{11}\ cm^{-2}$. Explicit calculations demonstrate that results for charged neutral impurity scattering are very similar [11]. Finally, the electron radiative decay $\Gamma$, is interpreted as coherent dipole re-radiation of the oscillating 2D electrons excited by microwaves; hence, it is given by $\Gamma = ne^2/\,(6\epsilon_0 c\, m^*)$, using the values of $n$ and $m^*$ given above, it yields $\Gamma \approx 0.38\, meV$.

One of the puzzling properties of the observed giant magnetoresistance oscillations is related to the fact that they appear only in samples with an electron mobility exceeding $\approx 1.5 \times 10^7 cm^2/Vs$. This phenomenon is absent in samples in which $\mu$ is reduced by one-order of magnitude. This behavior is well reproduced by the present formalism. Figure (1) displays the $\rho_{xx}\ vs.\ \omega/\omega_c$ plot for three selected values of $\mu$. For $\mu \approx 0.5 \times 10^7 cm^2/Vs$ the expected almost linear behavior $\rho_{xx} \propto B$ is clearly depicted. As the electron mobility increases to $\mu \approx 1.5 \times 10^7 cm^2/Vs$ the magnetoresistance oscillations are clearly observed, but negative resistance states only appear when $\mu \approx 2.5 \times 10^7 cm^2/Vs$. We notice that $\rho_{xx}$ vanishes at $\omega/\omega_c = j$ for $j$ integer. The oscillations follow a pattern with minima at $\omega/\omega_c = j + \epsilon$, and maxima at $\omega/\omega_c = j - \epsilon$, adjusted with $\epsilon \approx 1/10$.

Eqs. (5-7) contain the main ingredients that explain the huge increase observed in the longitudinal conductance when the material is irradiated by microwaves. In the standard expression for the Kubo formula there are no Floquet replica contribution, hence $\omega$ can be set to zero in (7), in which case $B^{(l)}$ becomes proportional to the energy derivative of the Fermi distribution, that in the $T \to 0$ limit becomes of the form $\delta(\epsilon - \epsilon_F)$, and the conductivity depends only on the states at the Fermi energy. On the other hand as a result of the periodic structure induced by the microwave radiation, $B^{(l)}$ is dominated by terms proportional to $\frac{d}{d\epsilon}A(\epsilon - \epsilon_\nu + l\omega)$, hence several contributions arise from the transitions between different Landau levels.

The model can also be used in order to test quirality effects induced by the magnetic field. In reference [11] we carried out calculations for different $E-$field polarizations with respect to the current. The amplitudes of the resistivity oscillation are bigger for transverse polarization as compared to longitudinal polarization. Selecting negative circular polarization, the oscillation amplitudes get the maximum possible value. Instead, the negative resistance states disappear

for positive circular polarization. These results are easily understood, because for negative circular polarization and $\omega \approx \omega_c$ the electric field rotates in phase with respect to the electron cyclotron rotation. Finally, we mention that the above discussed results are well described by single photon processes, however the present formalism is well suited to explore the non-linear regime in which multiple photon exchange play an essential role.

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